Vidyamandir Classes
Gurukul for IITJEE & Medical Preparation

SOLUTIONS

Joint Entrance Exam | IITJEE-2019

12th APR 2019 | Evening Session

Joint Entrance Exam | JEE Mains 2019

PART-A	PHYSICS

1.(3) $u = b\sqrt{x}$

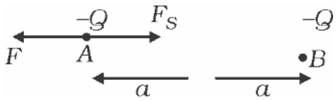
$\Rightarrow \quad \frac{dx}{dt} = b\sqrt{x} \quad \Rightarrow \quad \int_0^x \frac{dx}{\sqrt{x}} = b\int_0^t dt$

$\Rightarrow \quad 2\sqrt{x} = bt \quad \Rightarrow \quad x = \frac{b^2t^2}{4}$

$\Rightarrow \quad u = \frac{dx}{dt} = \frac{b^2t}{2} \quad \Rightarrow$ velocity at $t = \tau$ is $v = \frac{b^2\tau}{2}$

2.(3) Consider force on A due to B

$$|F| = \frac{Q^2}{4\pi\epsilon_0(2a)^2}$$



$$= \frac{Q^2}{16\pi\epsilon_0 a^2}$$

Since A is in equilibrium, force on it due to changed sphere, $F_s = F$ (magnitude)

So, electric field at distance a from centre

$$E = \frac{Q}{16\pi\epsilon_0 a^2}$$

If $r > R,$ result for electric field is $E = \frac{2Q}{4\pi\epsilon_0 r^2}$

So, assuming $a > R,$

$$E = \frac{Q}{16\pi\epsilon_0 a^2} = \frac{2Q}{4\pi\epsilon_0 r^2}$$

$\Rightarrow \quad r = 2\sqrt{2}a \quad \Rightarrow \quad a = \frac{v}{2\sqrt{2}}$

But the result was only valid for $r > R$

$\Rightarrow$ The point is inside the sphere and $a < R$

Let electric field inside the sphere at distance r from centre be E, then by gauss law

$$E(4\pi r^2) = \frac{1}{\epsilon_0}\int_0^r kx(4\pi x^2)dx$$

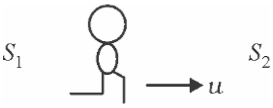
$\Rightarrow \quad E = \frac{kr^2}{4\epsilon_0}$

Also $\int_0^R kx(4\pi x^2)dx = 2Q \quad \Rightarrow \quad k = \frac{2Q}{\pi R^4}$

$\Rightarrow \quad E = \frac{Qr^2}{2\pi\epsilon_0 R^4} = \frac{Qa^2}{2\pi\epsilon_0 R^4}$

Comparing with $E = \frac{Q}{16\pi\epsilon_0 a^2} \quad \Rightarrow \quad a = 8^{-1/4}R$

3.(4)



$f_1 = f_0\left(\frac{v-u}{v}\right)$ and $f_2 = f_0\left(\frac{v+u}{v}\right)$

$f_2 - f_1 = \frac{f_0}{v}2u \quad \Rightarrow \quad u = \frac{v(f_2 - f_1)}{2f_0} = \frac{330\times 10}{2\times 660} = 2.5\text{m/s}$

4.(2) $\frac{1}{6} = 1 - \frac{T_2}{T_1} \Rightarrow \frac{T_2}{T_1} = \frac{5}{6}$ … (1)

$\frac{1}{3} = 1 - \frac{(T_2 - 62)}{T_1} \Rightarrow \left(\frac{T_2 - 62}{T_1}\right) = \frac{2}{3}$ … (2)

From (1) and (2)

$\frac{1}{6} = \frac{62}{T_1}$

$\Rightarrow \quad T_1 = 372k \quad \text{or} \quad 99°\text{C}$

$T_2 = \frac{5}{6}\times 372 = 5\times 62 \ = 310$ K or 37°C

5.(1) Coordinates of the masses :

$m_1 : (0,\ 0) \qquad m_2 : (1,\ 0)$

$m_3 : (10.5,\ 0.5\sqrt{3})$

$X_{CM} = \frac{50(0)+100(1)+150(0.5)}{50+100+150} = \frac{7}{12}m$

$Y_{CM} = \frac{50(0)+100(0)+150(0.5\sqrt{3})}{50+100+150} = \frac{\sqrt{3}}{4}m$

Coordinates of CM : $\left(\frac{7}{12}, \frac{\sqrt{3}}{4}\right)$

6.(1) $n(r) = n_0 e^{-\alpha r^4}$

Molecules in shell element of radii r and $r + dr$

$dN = n(r)dV \ = n(r)(4\pi r^2 dr)$

Total molecules $= N = \int dN$

$\Rightarrow \quad N = 4\pi n_0 \int r^2 e^{-\alpha r^4} dr$

Put $\quad \alpha r^4 = t \quad \Rightarrow \quad 4\alpha r^3 dr = dt$

$\Rightarrow \quad r^2 dr = \frac{dt}{\alpha r} \ = \alpha^{-3/4} t^{-1/4} dt$

$\Rightarrow \quad N = 4\pi n_0 \alpha^{-3/4} \int t^{-1/4} e^{-t} dt$

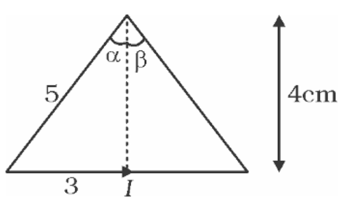
$\Rightarrow \quad n \propto n_0 \alpha^{-3/4}$

7.(4) $B = \frac{\mu_0 I}{4\pi r}(\sin\alpha + \sin\beta)$

$r = 4\,\text{cm}$

$\sin\alpha = \sin\beta = 3/5$

$\Rightarrow \quad \beta = 1.5\times 10^{-5}T$



8.(4) Let initially, there are n nuclei of each type.

$$n_A = ne^{-\left(\frac{t}{t_{1/2}}\right)\ell n 2} = ne^{-\frac{t\ell n2}{10}}$$

$$n_B = ne^{\frac{-t\ell n2}{20}}$$

At $t = 60$ minutes,

$n_A = \frac{n}{2^6}$ and $n_B = \frac{n}{2^3}$

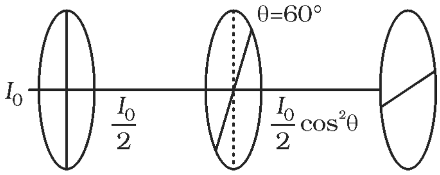
Ratio of decayed numbers $= \frac{n - n_A}{n - n_B} = \frac{n - \frac{n}{2^6}}{n - \frac{n}{2^3}} = \left(\frac{9}{8}\right)$

9.(1) 

Strain $= \frac{\Delta L}{L} = \frac{\alpha L \Delta\theta}{L}$

$Y = \frac{F}{A\alpha\Delta\theta} \quad \Rightarrow \quad \alpha = \frac{F}{Y\pi r^2 T}$

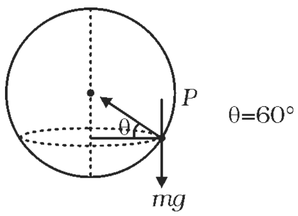
$$\left(Y = \frac{3F}{Y\pi r^2 T}\right)$$

10.(1) 

$\frac{I_0}{2}\cos^2\theta \cdot \cos^2 30° = I$

$\frac{I_0}{I} = \frac{2}{\frac{1}{4} \times \frac{3}{4}} = \frac{32}{3} = 10.67$

11.(3) $N\cos\theta = m\left(\frac{r}{2}\right)\omega^2$



$N\sin\theta = mg$

$\cot\theta = \frac{r\omega^2}{2g}$

$\omega = \frac{2g\cot\theta}{r} = \sqrt{\frac{2g}{r}\sqrt{3}}$

$\omega^2 = \frac{2g\sqrt{3}}{r}$

12.(3) $\beta = 10\log_{10}\left(\frac{P}{4\pi r^2 I_0}\right)$

$120 = 10\log_{10}\left(\frac{P}{4\pi r^2 I_0}\right)$

$\Rightarrow \quad \frac{P}{4\pi r^2 I_0} = 10^{12} \quad \Rightarrow \quad P = 10^{-12} \times 10^{12} \times 4 \times \pi r^2$

$\Rightarrow \quad r = \sqrt{\frac{P}{4\pi}} = \sqrt{\frac{2\times 7}{4\times 22}} = \sqrt{\frac{7}{44}} \approx 0.4 \quad r = 40\,\text{cm}$

13.(4) $nR\Delta T = 10$

$\Delta 4 = nC_v\Delta T = ?$ $\Rightarrow$ $\frac{10}{R}\times\frac{5}{2}R = \frac{50}{2}J$ $\left\{C_v = \frac{5R}{2}\right\}$

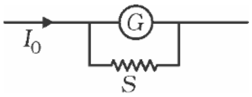
Hence $Q = \Delta u + W$

$Q = 35J$

14.(1) $q = \int_0^{\tau}\frac{E}{R}\left(1 - e^{-\frac{tR}{L}}\right)dt$

$$= \left[\frac{E}{R}t - \left[\frac{E}{R}e^{-\frac{tR}{L}}\right]\frac{L}{R}\right]_0^{\tau} = \frac{E}{R}\tau + \frac{EL}{R^2}e^{-1} - \frac{EL}{R^2} = \frac{EL}{R^2} + \frac{EL}{eR^2} - \frac{EL}{R^2} = \frac{EL}{eR^2} = \frac{EL}{(2.7)R^2} \qquad e = 2.7$$

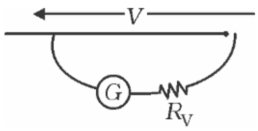
15.(1) **Ammeter**



$(I_0 - I_G)R_A = I_G G$ (1) $\Rightarrow$ $I_0 R_A = I_g(R_A + G)$

Voltmeter

$V = I_g(R_v + G)$ (2)



$\Rightarrow$ $V - I_g G = I_g R_V$ (3)

$G(I_0 - I_g) = I_g R_v$

$(I_0 - I_g)R_A = I_g G$

Dividing

$\frac{G}{R_A} = \frac{R_v}{G}$ $\Rightarrow$ $R_v R_A = G^2$

From (1) $R_A = \frac{I_g \cdot G}{I_0 - I_g}$

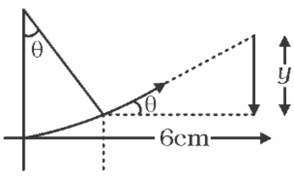
From (3) $R_v = \frac{G(I_0 - I_g)}{I_g}$

$\frac{R_A}{R_v} = \frac{I_g^2}{(I_0 - I_g)^2}$

16.(4) $g = \frac{GM}{R^2}$

$\frac{4}{g} = \frac{g_P}{g_E} = \frac{1}{9}\frac{R_E}{R_P}$ $\Rightarrow$ $R_p^2 = \frac{R^2}{4}$ So, $R_p = \frac{R}{2}$

17.(4)



$$R = \frac{\sqrt{2m\times 100eV}}{eB} = \sqrt{\frac{2\times 9.1\times 10^{-31}\times 6.6\times 10^{-17}}{1.6\times 10^{-19}\times 1.5\times 10^{-3}}}$$

$$= \frac{\sqrt{29.12}\ \ 10^{-24}}{1.6\times 1.5\times 10^{-22}} \approx 2.25\text{cm}$$

$\sin\theta = \frac{2cm}{R} = \frac{2\times 4}{9}$

$\sin = \frac{8}{9}$

$y = 6\tan\theta$ $= 6\times\frac{8}{\sqrt{17}} \approx 12\text{cm}$

So $d \approx 12.87\text{cm}$ $\{d = y + \Delta\}$

18.(4) $\lambda = 2(\ell_2 - \ell_1)$ (Resonance column exp.)

$= 2 \times 40\text{cm}$

$V = 480 \times 80 \;\; = 384\text{m/s}$

19.(3) Spring first $\alpha \dfrac{1}{\text{length}} \quad \Rightarrow \quad \left(\dfrac{k_1}{k_2} = \dfrac{1}{n}\right)$

or $\quad \dfrac{l_1}{l_2} = \dfrac{1}{n} \quad \Rightarrow \quad l_1 = \dfrac{l}{n+1}$

and $\quad l_2 = \dfrac{nl}{(n+1)}$

$k_1 = (n+1)k$

$k_2 = \dfrac{k(n+1)}{n} \quad \Rightarrow \quad \dfrac{k_1}{k_2} = \dfrac{1}{n}$

20.(4)

(A)	**(B)**
$f_1 = \mu(mg + F\sin\theta)$	$f_2 = \mu(mg - f\sin\theta)$
$a_1 = \dfrac{F.\cos 30° - f_1}{m}$	$a_2 \dfrac{F\cos 30° - f_2}{m}$
$a_1 = \dfrac{F\cos\theta - \mu mg - \mu F\sin\theta}{m}$	$a_2 = \dfrac{F\cos - \mu mg + \mu F\sin\theta}{m}$

$a_2 - a_1 = \dfrac{2\mu F\sin\theta}{m} \quad = \dfrac{2 \times 0.2 \times 20 \times 1}{5 \times 2} \;\; = 0.8 \text{ m/s}^2$

21.(4) Direction of propagation + z-axis

$B_0 = \dfrac{E_0}{C} = 2 \times 10^{-7}\,T$

So only choice option (4)

22.(3) $mVr = \dfrac{3h}{2\pi}(n = 3)$

$mV = \dfrac{3h}{2\pi \times r} = \dfrac{3h}{2 \cdot \pi r}$

$\lambda = \dfrac{h}{mV} = \dfrac{2 \cdot h\ \pi r}{3 \cdot h} \;\; = \dfrac{2\pi r}{3} \;\; = \dfrac{2}{3} \times 4.65\text{Å} \;\; = 9.7\text{Å}$

23.(3) $(ms\Delta\theta) + mL = l_{rms}^2 Rt$

$\Rightarrow \quad t = \left(\dfrac{ms\Delta\theta + mL}{l_{rms}^2 R}\right)$

$t \approx 22$ minutes

24.(4) $V_z = 6V$

$V = 1R_s + V_z \qquad \{V_{mass} = 16\text{volt}\}$

$16 = 1R_s + 6$

$iR_s = 10 \qquad i = \dfrac{10}{2 \times 10^3} = 5mA$

This current will be divided

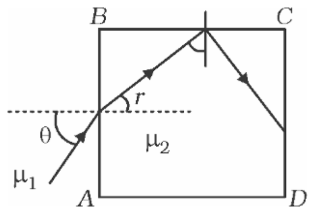
$V_z = I_L R_L$

$\dfrac{6}{4} = I_L$

$\Rightarrow \quad i_L = 1.5mA$

So current through Zener = 3.5 mA

25.(2) $90 - r \geq \theta_C$



$$90 - r \geq \sin^{-1}\left(\frac{\mu_1}{\mu_2}\right) \Rightarrow \cos r \geq \frac{\mu_1}{\mu_2}$$

Snell's law

$$\frac{\sin\theta}{\sin r} = \frac{\mu_2}{\mu_1} \Rightarrow \sin\theta = \frac{\mu_2}{\mu_1}\sin r = \frac{\mu_2}{\mu_1}\sqrt{1-\left(\frac{\mu_1}{\mu_2}\right)^2}$$

$$\sin\theta = \sqrt{\left(\frac{\mu_2}{\mu_1}\right)^2 - 1}$$

This is maximum θ so $\theta \leq \sqrt{\left(\frac{\mu_2}{\mu_1}\right)^2 - 1}$

26.(3) Terminal velocity for sphere of radius r

$$v = \frac{2}{9\eta}(d-\rho)r^2 g$$

$$\Rightarrow \quad v \propto r^2$$

$$r_2^3 = \frac{r_1^3}{27} \Rightarrow \frac{r_1}{r_2} = 3 \Rightarrow \frac{v_1}{v_2} = \left(\frac{r_1}{r_2}\right)^2 = 9$$

27.(4) L.S.B $= \omega_c - \omega_m = (20000 - 2000)\pi = 18000\pi$ rad/s

So $\frac{18000\pi}{2\pi} = 9000Hz$

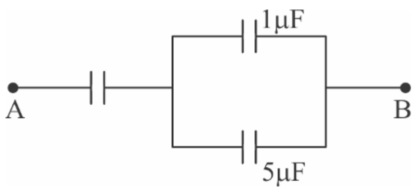
Modulation index $= \frac{A_m}{A_c} = 0.5$

28.(4) Two angles of projection are α and β

Range is equal $\Rightarrow \alpha + \beta = 90°$

$$h_1h_2 = \left(\frac{u^2\sin^2\alpha}{2g}\right)\left(\frac{u^2\sin^2\beta}{2g}\right) = \frac{u^4\sin^2\alpha\cos^2\alpha}{4g^2} = \frac{u^4\sin^2 2\alpha}{16g^2} = \frac{1}{16}\left(\frac{u^2\sin 2\alpha}{g}\right)^2 = \frac{R^2}{16} \Rightarrow R^2 = 16\,h_1h_2$$

29.(4) $V_A = V_B = 10V$

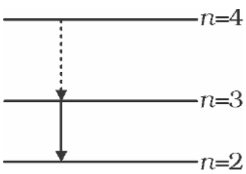


In series, potential difference across C_1 is $\frac{C_2V}{C_1+C_2}$

$\Rightarrow$ Potential difference across $4\mu F = \frac{6}{4+6}\times 10 = 6V$

$\Rightarrow$ Charge $= 4\times 6 = 24\,\mu C$

30.(4)



$$h_c/\lambda_1 = \frac{Rhc\times 7}{16\times 9}$$

$$h_c/\lambda_2 = \frac{Rhc\times 5}{4\times 9}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{7\times 4\times 9}{16\times 9\times 5} = \frac{7}{20}$$

$$\left(\frac{\lambda_1}{\lambda_2} = \frac{20}{7}\right)$$

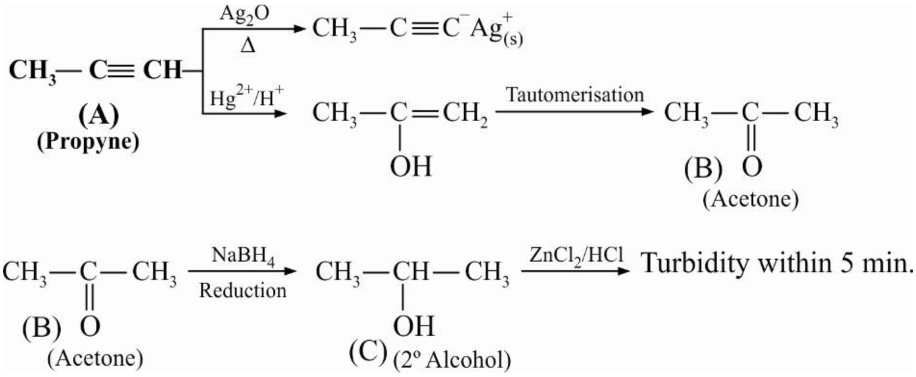
PART-B	CHEMISTRY

1.(1) Energy of 2s orbital $\propto \frac{1}{(Z_{eff})^2}$

$$Z_{eff} : K = Na > Li > H \quad \& \quad Z : K > Na$$

hence the force of attraction of the K nuclei at 2s electrons will be stronger than that of Na nuclei.

2.(2)



3.(2) $\underset{(X)}{2KMnO_4} \xrightarrow{\Delta} \underset{(Y)}{K_2MnO_4} + MnO_2 + \frac{1}{2}O_2$

$$MnO_2 + 2Cl^- + 4H^+ \longrightarrow Mn^{+2} + 2H_2O + Cl_2$$

4.(2) $Cd(OH)_2 \rightleftharpoons Cd^{+2} + 2OH^-$

$K_{sp} = [Cd^{+2}][OH^-]^2$

solubility $s = [Cd^{+2}] = \frac{K_{sp}}{[OH^-]^2}$

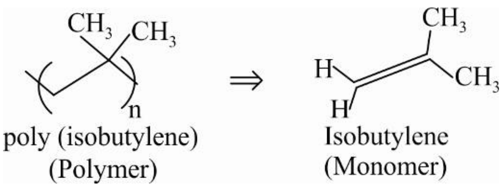
pH of the buffer = 12

pOH = 2, $[OH] = 10^{-2}$

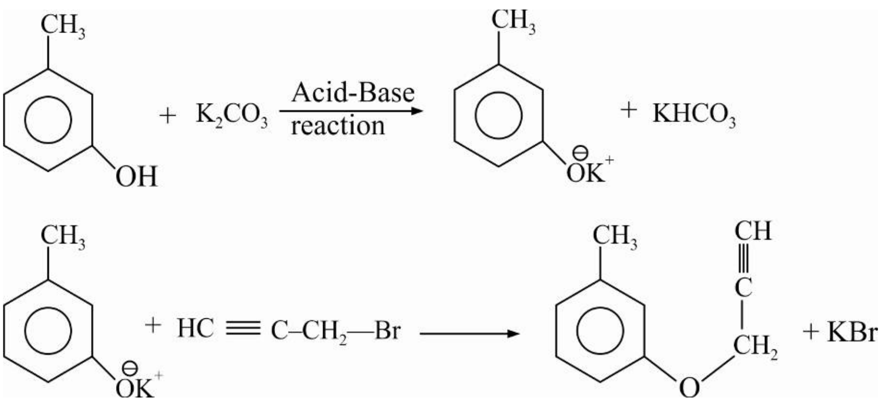
$K_{sp} = 4s^3 = 4(1.84 \times 10^{-5})^3$

$$[Cd^{+2}]_f = \frac{4[1.84 \times 10^{-5}]^3}{(10^{-4})} = \frac{4 \times 6.23 \times 10^{-15}}{10^{-4}} = 2.49 \times 10^{-10}\,M.$$

5.(1)

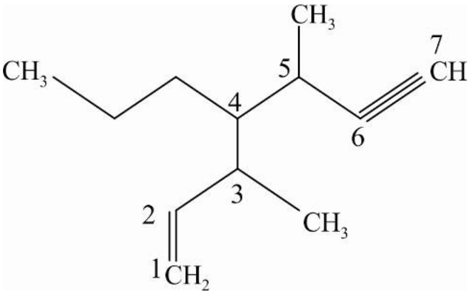


6.(2)



7.(1) Mo and W have nearly same atomic radii due to lanthanide contraction.

8.(2)



3, 5-dimethyl-4-propyl hept-1-en-6-yne.

9.(4) EDTA is used to remove lead poisoning by formation of stable $Pb(EDTA)^{2-}$ complex.

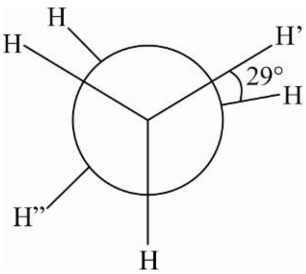
10.(1) Primary pollutant in the photochemical smog: **NO_2 and Hydrocarbons**.
Secondary pollutant in the photochemical smog: **Ozone and acrolein (PAN).**

11.(2) The osmotic pressure of the colloidal solutions is generally LOWER than a true solution of same. Concentration due to chances of association of the particles.

12.(2) If $\Delta G^\circ < O$ then $K_{eql} > 1$

13.(4) In Vinyl Halides $CH_2 \overset{a}{=} CH \overset{b}{-} X$, the strength of the bond (b) is very high due to delocalisation such that the Vinyl carbocation is highly unstable.

14.(2)



Dihedral angle $= 120^\circ + 29^\circ = 149^\circ$

15.(4) C – C bond length will be maximum in diamond due to sp^3 Hybrid state of the carbon atoms.

16.(3) Rate of disappearance of $N_2O_5 = \frac{(3-2.75)}{30} = \frac{25}{30} \times 10^{-2}\ M\ min^{-1} = \left(\frac{5}{6}\right) \times 10^{-2}\ M\,min^{-1}$

Rate of formation of $NO_2 = 2$ (rate of disappearance of N_2O_5) $= \frac{10}{6} \times 10^{-2} = \mathbf{1.667 \times 10^{-2}\ Mmin^{-1}}$

17.(2) $\pi = \pi_1 + \pi_2$

$\pi_1 =$ osmotic pressure of urea solution.

$\pi_2 =$ osmotic pressure of glucose solution.

$\pi_1 = C_1RT \quad \pi_2 = C_2RT$

$C_1 = \frac{0.6/60}{0.1} = 0.1\,M$

$C_2 = \frac{1.8/180}{0.1} = 0.1\,M$

$\pi = \pi_1 + \pi_2 = 0.1\,RT + 0.1\,RT$

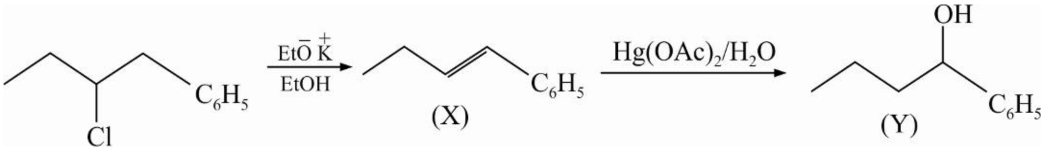
$\pi = 2(0.1\,RT) = 0.2 \times 24.6 = 4.92$ atm.

18.(2) $2NO_{(g)} \quad N_{2(g)} + O_{2(g)},\ \Delta n = 0\ \ (K_P = K_C)$

$2C_{(s)} + O_{2(g)} \quad 2CO_{(g)}, \Delta n = +1\ (K_P > K_C)$

$2HI_{(g)} \quad H_{2(g)} + I_{2(g)}, \Delta n = 0\ (K_P = K_C)$

$NO_{2(g)} + SO_{2(g)} \quad NO_{(g)} + SO_3,\ \Delta n = 0\ \ (K_P = K_C)$

19.(4) 

20.(3) Nuclear charge : Be < B

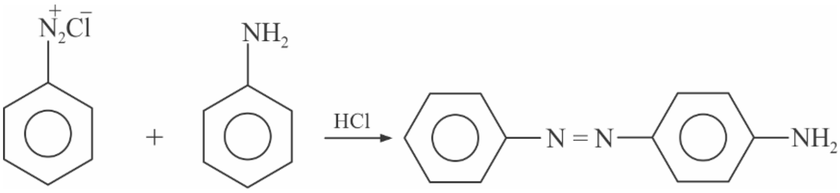
First ionization energy : Be > B (due to higher stability of $2s^2$ configuration over $2s^2 2p^1$)

21.(4) No. of ions $\quad HCOOH_{aq} > C_6H_5COOH > CH_3COOH_{aq.}$

(A) (C) (B)

Conductivity $\propto$ no. of ions.

$\Rightarrow$ A > C > B

22.(3) 

23.(2) $(CH_3)_3C - Cl + AgNO_3 \longrightarrow AgCl\downarrow + (CH_3)_3C^+NO_3^-$

24.(2) $Al_2O_3 + \text{impurity of } SiO_2 \xrightarrow[\Delta]{NaOH_{aq}} Na[Al(OH)_4] + Na_2SiO_3$ [Hall-Heroult's process]

25.(1) $Mg(HCO_3)_2 \xrightarrow{\Delta} MgCO_{3aq} + CO_{2(g)} + H_2O_{(l)}$

$MgCO_{3\,aq.} \xrightarrow[\text{Hydrolysis}]{\Delta} \underset{\text{(ppt.)}}{Mg(OH)_2} \downarrow + CO_{2(g)} + H_2O$

($Mg(OH)_2$ is less soluble than $MgCO_3$ in aqueous medium)

26.(4) $C_xH_y + (x + y/4)O_2 \longrightarrow x\,CO_2 \ + \ y/2\ H_2O$

25 g $\qquad$ 88 g $\quad$ 9 gm

2 moles $\quad$ 1 / 2 mole

(Molar ratio of C & H in Hydrocarbon)

$$\frac{x}{y/2} = \frac{2}{1/2}$$

$$\frac{2x}{y} = \frac{4}{1}$$

$$\therefore\ \frac{x}{y} = \frac{2}{1}$$

Hence, 25 g of the Hydrocarbon have 24 g of the Carbon and 1 g of the Hydrogen

27.(4) $2Li\,NO_3 \xrightarrow{\Delta} Li_2O_{(s)} + 2NO_{2(g)} + O_{2(g)}\uparrow$

(Lithium nitrate)

28.(3) $[Co\,Cl\,(en)_2]\,Cl$ $\quad$ $K_3[Al\ (C_2O_4)_3]$
$(C.N. = 5)$ $\quad$ $(C.N. = 6)$

29.(1) Glycogen is NOT similar in structure to amylose which is a straight chain polymer. It shows similarity with amylopectin in its structure with higher number of branches. Glycogen is generally found in animal cells and also in some yeast and fungi. It only have $\alpha-$ linkages between the monomeric units.

30.(4) Simple cubic = 1
B. C. C. = 2
F.C.C. = 4 ratio $\Rightarrow$ 1 : 2 : 4

PART-C	MATHEMATICS

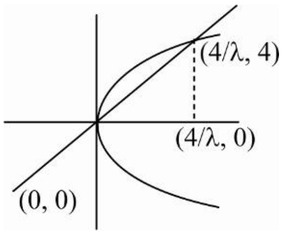
1.(4) $$\int_{\alpha}^{\alpha+1} \frac{dx}{(x+\alpha)(x+\alpha+1)} = \int_{\alpha}^{\alpha+1}\left(\frac{1}{(x+\alpha)} - \frac{1}{(x+\alpha+1)}\right)$$

$$= \ln\left(\frac{x+\alpha}{x+\alpha+1}\right)\Bigg]_{\alpha}^{\alpha+1} = \ln\left(\frac{2\alpha+1}{2\alpha+2}\right) - \ln\left(\frac{2\alpha}{2\alpha+1}\right) = \ln\left(\frac{(2\alpha+1)^2}{2(\alpha)(\alpha+1)}\right) = \ln\frac{9}{8} \Rightarrow \alpha = -2$$

2.(2) $$\lim_{x\to 0} \frac{\left(\frac{x}{x}+\frac{2\sin x}{x}\right)\left(\sqrt{x^2+2\sin x+1}+\sqrt{\sin^2 x - x + 1}\right)}{\frac{x^2}{x} - \frac{\sin^2 x}{x} + \frac{2\sin x}{x} + \frac{x}{x}} = 2$$

3.(2) $y^2 = 4\lambda x,\ y = \lambda x$

$$A = \int_0^{4/\lambda} (2\sqrt{\lambda x} - \lambda x)\, dx \Rightarrow A = \frac{\frac{2\sqrt{\lambda}8}{\lambda^{3/2}}}{\frac{3}{2}} - \frac{\lambda \times \frac{16}{\lambda^2}}{2} = \frac{1}{9} \Rightarrow \lambda = 24$$



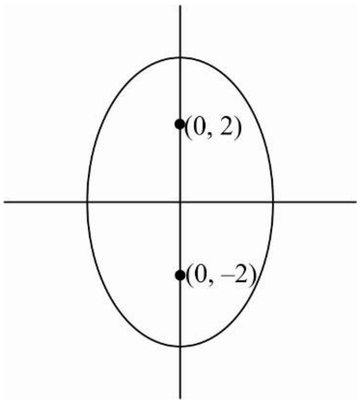
4.(4) $2be = 4$, $a = 2$

$b^2e^2 = 4$

$b^2 - a^2 = 4$

$b^2 = 8$

$\frac{x^2}{4} + \frac{y^2}{8} = 1$



5.(1) Normal to plane : $(\hat{i} + 2\hat{j} - \hat{k}) \times (-\hat{i} + \hat{j} - 2\hat{k}) = -3i + 3\hat{j} + 3\hat{k}$
Equation of plane is $-x + y + z = 0$

$$P = \frac{-2+1+4}{\sqrt{3}} = \sqrt{3}$$

6.(1) $A = \{1, 2, 3, 4\}$
$B = \{1, 2, 4\}$

$C = \{1, 2, 3, 4\}$ Not satisfying option (1)

7.(2) $xy\dfrac{dy}{dx} = (y^2 - x^3)$

$y\dfrac{dy}{dx} - \dfrac{y^2}{x} = -x^2$

$y^2 = t \Rightarrow 2y\dfrac{dy}{dx} = \dfrac{dt}{dx}$

$2\dfrac{dt}{dx} - \dfrac{t}{x} = -x^2 \Rightarrow \dfrac{dt}{dx} - \dfrac{2}{x}t = -2x^2$

$IF = e^{-\int \frac{1}{x}dx} = e^{-2\ln(x)} = \dfrac{1}{x^2}$

$\dfrac{t}{x^2} = \int -2\,dx$

$\dfrac{t}{x^2} = -2x + C \Rightarrow y^2 = -2x^3 + Cx^2$

8.(3) $\dfrac{2x - y + 2z - 4}{3} = \pm\dfrac{(x + 2y + 2z - 2)}{3}$

$x - 3y = 2$, $3x + y + 4z = 6$

9.(4) $\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$

$C_1 \to C_1 + C_2$ $R_1 \to R_1 - R_2$

$\begin{vmatrix} 2 & \sin^2\theta & 4\cos 6\theta \\ 2 & 1+\sin^2\theta & 4\cos 6\theta \\ 1 & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 0 & -1 & 0 \\ 2 & 1+\sin^2\theta & 4\cos 6\theta \\ 1 & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$

$(2 + 8\cos 6\theta) - 4\cos 6\theta = 0$

$2 + 4\cos 6\theta = -\dfrac{1}{2}$

$6\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$, $\theta = \dfrac{\pi}{9}, \dfrac{2\pi}{9}$

10.(1) $\beta^2 = \alpha\gamma$, $\beta = \sqrt{\alpha\gamma}$

$\left(\sqrt{\alpha}x + \sqrt{\gamma}\right)^2 = 0 \Rightarrow x = -\dfrac{\sqrt{\gamma}}{\sqrt{\alpha}}$ is a root of $x^2 + x - 1 = 0 \Rightarrow \dfrac{\gamma}{\alpha} - \dfrac{\sqrt{\gamma}}{\sqrt{\alpha}} - 1 = 0$

$\Rightarrow \dfrac{\gamma - \sqrt{\alpha\gamma}}{\alpha} = 1 \Rightarrow \dfrac{(\gamma - \beta)}{\alpha} = 1 \Rightarrow \gamma = (\beta + \alpha)$

$\alpha\beta + \alpha\gamma = \alpha\beta + \beta^2 = \beta(\alpha + \beta) = \beta\gamma$

11.(4) $3a + 6d + 15d = 40$

$3a + 21d = 40$

$a + 7d = \dfrac{40}{3}$

$S_{15} = \dfrac{15}{2}(2a + 14d) = 15 \times \dfrac{40}{3} = 200$

12.(4)

X	**15**	**12**	**–6**

$P(X)$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{26}{36}$

Expectation $=\frac{15}{6}+\frac{12}{9}-6\times\frac{26}{36} \quad = \quad \frac{15+8-26}{6}=-\frac{1}{2}$

13.(2) $7+\cos 2x=\alpha(2-\sin x)$

$8-2\sin^2 x=\alpha(2-\sin x)$

$\alpha=2(2+\sin x)$

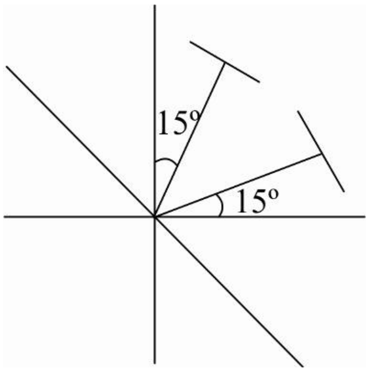
$\alpha\in[2,\ 6]$

14.(4) $\sum_{r=0}^{20} r^2\ {}^{20}C_r$

$S=20\sum_{r=1}^{20}(r-1+1)\ {}^{19}C_{r-1}\ =20\times 19\times\sum_{r=2}^{20}{}^{18}C_{r-2}+20\times\sum_{r=1}^{20}{}^{19}C_{r-1}$

$=20\times 19\times 2^{18}+20\times 2^{19}\ \ =20\times 2^{18}(19+2)\ \ =21\times 5\times 2^{20}=1052^{20}$

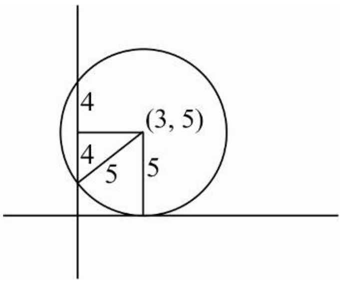
15.(1,3)



$x\cos 15^\circ+y\sin 15^\circ=4$

$x\cos 75^\circ+y\sin 75^\circ=4$

16.(2)



$(x-3)^2+(y-5)^2=25$

17.(3) $\begin{vmatrix}\alpha & 1 & 3\\ 2 & 1 & -\alpha\\ \alpha & -2 & 3\end{vmatrix}=0$

$\alpha(3-2\alpha)-1(6+\alpha^2)+3(-4-\alpha)=0$

$3\alpha-2\alpha^2-6\alpha^2-12-3\alpha=0$

$3\alpha^2=-18$

$\alpha^2=-6$

18.(1) $f(x)=\tan^{-1}\left(\frac{\tan x-1}{\tan x+1}\right)\ \Rightarrow\ f(x)=\tan^{-1}\tan\left(x-\frac{\pi}{4}\right)\quad x\in\left(0,\ \frac{\pi}{2}\right)\ \Rightarrow\ f(x)=x-\frac{\pi}{4}$

19.(3) $\alpha=2,\ \beta=-1$

$$\lim_{x\to 2}\frac{(x-1)(x-2)(x-3)}{(x-4)(x-2)}=\frac{-1}{-2}=\frac{1}{2}$$

20.(1) $\displaystyle\int\frac{\sin(x+\alpha)}{\sin(x-\alpha)}\,dx$

$x-\alpha=t\Rightarrow x=t+\alpha$

$\displaystyle\int\frac{\sin(t+2\alpha)}{\sin t}\,dt$

$\cos(2\alpha)t+\sin 2\alpha\ \ln|\sin t|+c$

21.(4) $boys=5,\ n=girls$

$boys$	$girls$
1	2
2	1

$^5C_1\times{}^nC_2+{}^5C_2\times{}^nC_1=1750$

$\dfrac{5n(n-1)}{2}+10n=1750$

$n^2-n+4n=700$

$n^2+3n=700$

$(n+28)(n-25)=0$, $\qquad n=25$

22.(2) $P(s)=\dfrac{4}{5},\ P(f)=\dfrac{1}{5}$

P(0 failure + 1 failure) $=\left(\dfrac{4}{5}\right)^{50}+{}^{50}C_1\times\left(\dfrac{4}{5}\right)^{49}\times\left(\dfrac{1}{5}\right)=\left(\dfrac{4}{5}\right)^{49}\left(\dfrac{4}{5}+10\right)=\left(\dfrac{4}{5}\right)^{49}\left(\dfrac{54}{5}\right)$

23.(3) $\sim(p\to(\sim q))\equiv\sim(\sim p\vee\sim q)\equiv(p\wedge q)$

24.(3) $\left(\dfrac{1}{60}-\dfrac{x^8}{81}\right)\left(2x^2-\dfrac{3}{x^2}\right)^6$

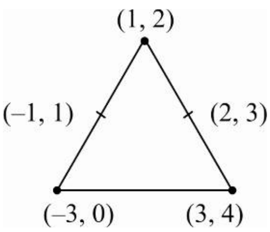
$T_{r+1}={}^6C_2(2x^2)^{6-r}\left(-\dfrac{3}{x^2}\right)^r={}^6C_r2^{6-r}(-3)^rx^{12-4r}$

Constant term in expansion of $\left(2x^2-\dfrac{3}{x^2}\right)^6={}^6C_2\,2^3(-3)^3$

Coefficient of x^{-8} in the expansion of $\left(2x^2-\dfrac{3}{x^2}\right)^6={}^6C_5(2)(-3)^5$

Term independent of $x=\dfrac{^6C_3\times2^3(-3)^3}{60}+\dfrac{^6C_5\times2\times3^5}{81}=-72+6\times6=-36$

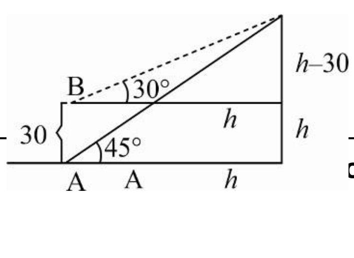
25.(2)



Centroid $=\left(\dfrac{1}{3},2\right)$

26.(2) $\dfrac{h-30}{h}=\dfrac{1}{\sqrt3}$

$\sqrt3h-h=30\sqrt3$



$$h = \frac{30\sqrt{3}}{(\sqrt{3}-1)} = \frac{30\sqrt{3}(\sqrt{3}+1)}{2} = 45(3+\sqrt{3})$$

27.(4) $[\sin\theta]x + [-\cos\theta]y = 0$

$[\cot\theta]x + y = 0$

$\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ $\qquad$ $\theta \in \left(\pi, \frac{7\pi}{6}\right)$

$0x + 0y = 0$ $\qquad$ $-x + 0y = 0$

$-x + y = 0$ $\qquad$ $[\cot\theta]x + y = 0$ and $[\cot\theta]$ is not identically sum integer

28.(1) $y^2 = 16x$ $\qquad$ $xy = -4$

$y = \left(mx + \frac{4}{m}\right)$ $\qquad$ $x\left(mx + \frac{4}{m}\right) = -4 \Rightarrow mx^2 + \frac{4}{m}x + 4 = 0$ has equal roots $(D = 0)$

$$\frac{16}{m^2} - 16m = 0 \Rightarrow m = 1$$

Common tangent $y = x + 4$

29.(4) $y = (x-2)^2 - 1$ and $x - y = 3$ intersect at (2, –1) and (3, 0)

$\left.\frac{dy}{dx}\right|_{x=2} = 0$, $\left.\frac{dy}{dx}\right|_{x=3} = 2$ equations of tangent is $y + 1 = 0$, $y = 2(x-3)$

Hence point of intersection $\left(\frac{5}{2}, -1\right)$

30.(3) $\frac{2z-n}{2z+n} = 2i - 1$

$\frac{(2x-n) + i20}{(2x+n) + i(20)} = (2i-1)$

$(2x-n) + i20 = (2i-1)(2x+n+i20) = -(2x+n) - 40 + (4x+2n)i - i20$

$-2x - n - 40 = 2x - n$ $\qquad$ $20 = (4x + 2n - 20)$

$4n = -40 \Rightarrow n = -10,$ $\qquad$ $20 = -60 + 2x \Rightarrow 2x = 80 \Rightarrow x = 40$